Abû Kâmil Worksheet (from his work *Algebra*)

1. Use Abû Kâmil's formula $\sqrt{a} \pm \sqrt{b} = \sqrt{a + b \pm 2\sqrt{ab}}$ for the difference of two square roots to show that $\sqrt{18} - \sqrt{8} = \sqrt{2}$, then express $\sqrt{18} + \sqrt{8}$ as a single square root.

2. Abû Kâmil was capable of solving equations containing irrational coefficients. For example, he solved the equation $(x + \sqrt{3})(x + \sqrt{2}) = 20$ by stating that

   $x = \sqrt{22\frac{1}{4} - \sqrt{6} + \sqrt{1\frac{1}{2}} - \sqrt{\frac{3}{4}} - \sqrt{\frac{1}{2}}}$. Show that his solution is correct.

3. Derive his solution to exercise #3 by completing the square on the quadratic equation $x^2 + (\sqrt{3} + \sqrt{2})x = 20 - \sqrt{6}$.

4. *Problem 15.* 10 dinar is divided equally among a group of men so that when 6 more men are added to their number and 40 is divided equally among them, then each receives as much as he did previously. Find the original number of men.

5. *Problem 19.* Given that 3 roots of a number plus 4 roots of the difference between the number and the 3 roots equals 20, find the number. [*Hint*: In the equation $3\sqrt{x} + 4\sqrt{x - 3\sqrt{x}} = 20$, let $x = y^2$ to obtain $20 - 3y = 4\sqrt{y^2 - 3y}$; then square both sides.]

6. *Problem 26.* Divide 10 into two parts in such a way that when a certain one of these parts is divided by the other and the fraction is multiplied by its numerator, the result gives 9.

7. *Problem 54.* Find a number such that if 7 is added to it and the sum multiplied by the root of 3 times the number, then the result is 10 times the number. [*Hint*: To solve the equation $(x + 7)\sqrt{3x} = 10x$, put $x = \frac{1}{3}y^2$; this yields $y^2 + 21 = 10y$.]

8. *Problem 66.* Divide 10 into two parts in such a way that when 50 is divided by one part and 40 by the other, and then the fractions are multiplied, 125 will result. [*Hint*: Algebraically, the equations are $x + y = 10$ and $\frac{50}{x} \cdot \frac{40}{y} = 125$.]