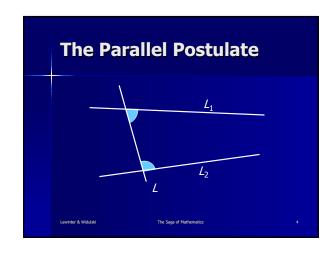


The Parallel Postulate If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.



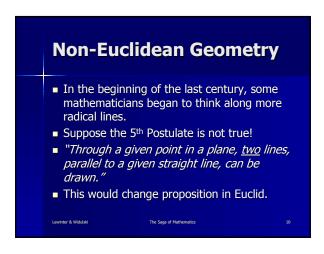
The Parallel Postulate The most convincing evidence of Euclid's mathematical genius. Euclid had no proof. In fact, no proof is possible, but he couldn't go further without this statement. Many have tried to prove it but failed.



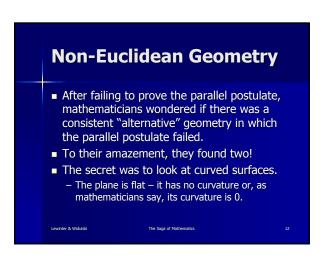


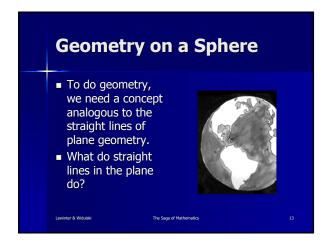


■ Given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line. ■ The Axiom is not Playfair's own invention. He proposed it about 200 years ago, but Proclus stated it some 1300 years earlier. ■ Often substituted for the fifth postulate because it is easier to remember.

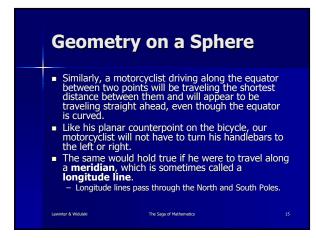


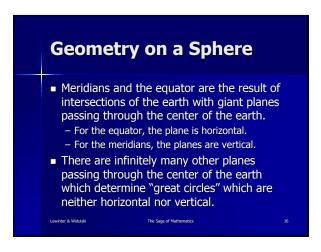
Non-Euclidean Geometry For example: The sum of the angles in a triangle is *less* than two right angles! This of course led to Non-Euclidean geometry whose discovery was led by Gauss (1777-1855), Lobachevskii (1792-1856) and Jonas Bólyai (1802-1850). Later by Beltrami (1835-1900), Hilbert (1862-1943) and Klein (1849-1945). Fits Einstein's *Theory of Relativity*.



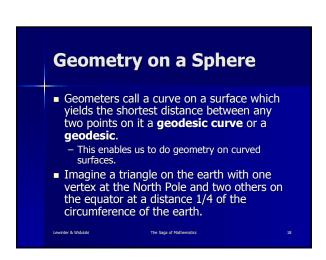


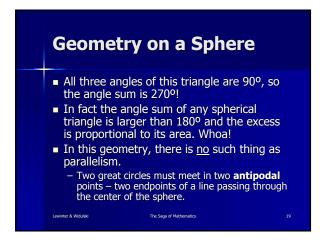


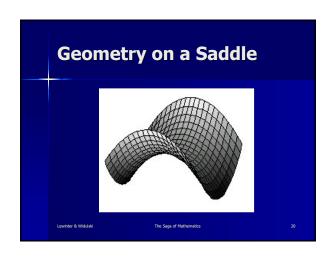




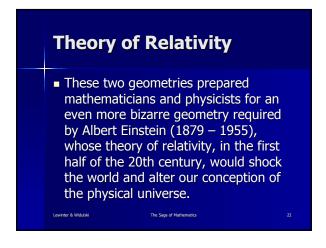
Geometry on a Sphere Given two points, such as New York City and London, the shortest route is not a latitude line but rather an arc of the great circle formed by intersecting the earth with a plane passing through New York, London and the center of the earth. This plane is unique since three noncollinear points in space determine a plane, in a manner analogous to the way two points in the plane determine a line.

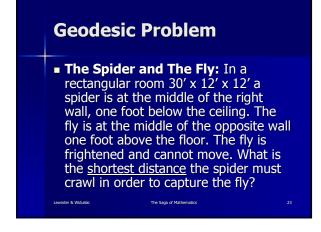


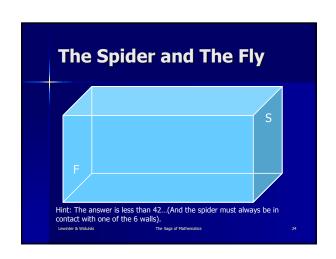


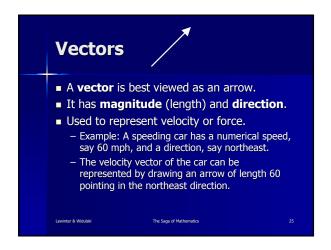


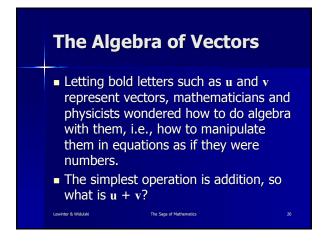
Geometry on a Saddle On this kind of surface, parallel geodesics actually diverge! They get farther apart, for example, if they go around different sides of its neck. The stranger part is that through a point P not on a given line L on the surface, there are infinitely many parallel lines. Furthermore, angle sums of triangles on these saddle-like surfaces are less than 180°.



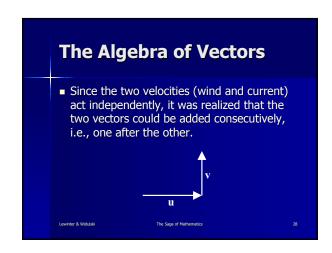


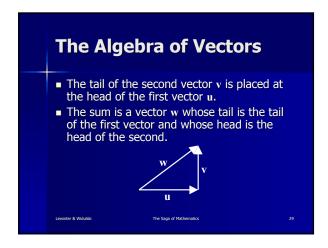


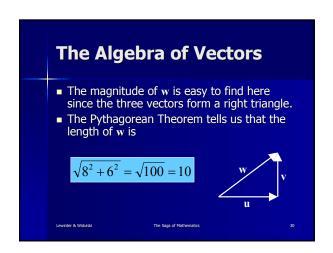




The Algebra of Vectors Picture yourself in a sailboat, and suppose the wind pushes you due east at 8 mph while the current pushes you due north at 6 mph. The sum of these vectors should reflect your actual velocity, including both magnitude and direction.

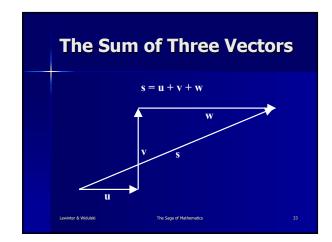


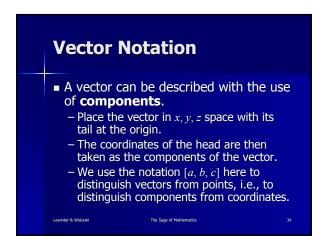




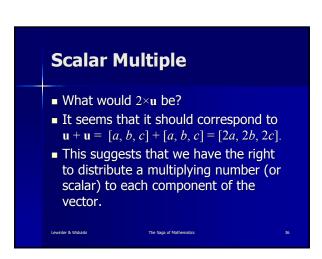
The Algebra of Vectors The speed of the sailboat is 10 mph. The boat is not traveling exactly northeast because the angle between vectors u and w is not 45°. The exact angle may be computed using trigonometry. It will be a bit less than 45° since v is shorter than u.

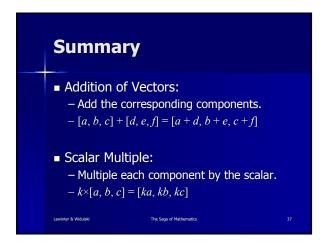
The Algebra of Vectors How do we evaluate sums of three or more vectors? The same way. Place them consecutively so that the tail of each vector coincides with the head of the previous one. The sum will be a vector whose tail is the tail of the first vector and whose head is the head of the last vector.

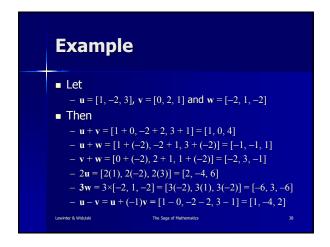




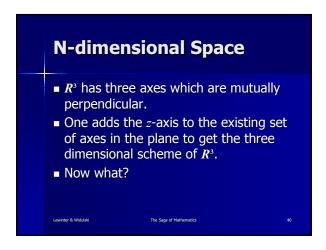
Addition of Vectors ■ Mathematicians were delighted to discover that the geometric instructions for addition given above simplify greatly to a mere adding of respective components. ■ Thus, if u = [a, b, c] and v = [d, e, f], then u + v = [a + d, b + e, c + f].



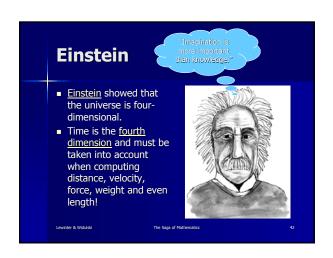




N-dimensional Space Mathematicians of the 19th century conjured up an *n*-dimensional world, called Rⁿ, in which points have *n* coordinates and vectors have *n* components! The above laws carry over quite easily to these *n*-dimensional vectors and yield an interesting theory which most find impossible to visualize. R² has two axes which are mutually perpendicular (meet at right angles).







Einstein [1879-1955]

- He posited that large massive objects (like our sun) curve the fourdimensional space around them and cause other objects to follow curved trajectories around them
 - hence the elliptic trajectory of the earth around the sun.
- Einstein correctly predicted that light bends in a gravitational field.

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Einstein [1879-1955]

- This was verified during a solar eclipse at a time when Mercury was on the other side of the sun and normally invisible to us.
- The eclipse, however, rendered it visible and it seemed to be in a slightly different location
 - Precisely accounted for by the bending of light in the gravitational field of the sun.

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The Saga of Mathematic

Mathematics is the Language of Science

- The 19th century was the time in which electromagnetic phenomena puzzled scientists.
- An electric current in a wire wrapped around a metal rod generated a magnetic field around it.
- On the other hand, a moving magnet generated a current in a wire.
- These phenomena are described by laws using vector fields.
 - Spaces in which each point is the tail of a vector whose magnitude and direction varies from point to point.

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The Speed of Light

- The speed of light was measured in two directions:
 - one in the direction of the motion of the earth
 - the other perpendicular to that direction
- The shocking fact was that both speeds were the same.
- It was finally realized that the speed of light seemed independent of the velocity of its
 - Contradicting the findings of Galileo and Newton.

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Galileo and Newton

- They posited the law of addition of velocities.
 - If a man on a train runs forward at 6 mph and if the train is moving at 60 mph, the speed of the man relative to an observer on the ground is

60 mph + 6 mph = 66 mph.

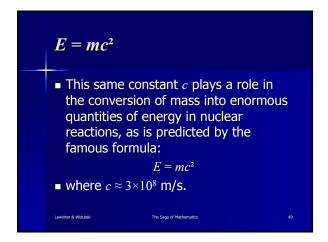
■ Why was light exempt from this law?

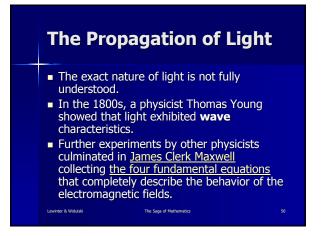
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The Speed of Light

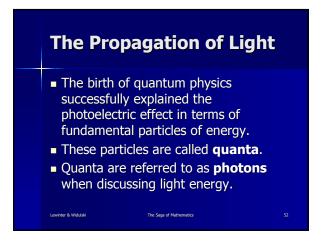
■ The answer emerged from Einstein's theory that the universe is four dimensional and requires a complicated mathematical scheme of calculation in which the velocity of light, i.e., the speed of propagation of electromagnetic energy, denoted c, is constant and is in fact the limiting speed of the universe.

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The Propagation of Light Maxwell deduced that light was simply a part of the electromagnetic spectrum. This seems to firmly establish that light is a wave. But, in the 1900s, the interaction of light with semiconductor materials, called the photoelectric effect, could not be explained by the electromagnetic-wave theory.



The Propagation of Light Today, when studying light that consists of many photons, as in propagation, that light behaves as a continuum - an electromagnetic wave. On the other hand, when studying the interaction of light with semiconductors, as in sources and detectors, the quantum physics approach is taken. The wave versus particle dilemma! Oh, no!

