Chapter 5

Must All Good Things
Come to an End?

Hypatia of Alexandria

- Born about 370 AD.
- She was the first woman to make a substantial contribution to the development of mathematics.
- She taught the philosophical ideas of Neoplatonism with a greater scientific emphasis.

Neoplatonism

- The founder of Neoplatonism was Plotinus.
- Iamblichus was a developer of Neoplatonism around 300 AD.
- Plotinus taught that there is an ultimate reality which is beyond the reach of thought or language.
- The object of life was to aim at this ultimate reality which could never be precisely described.

Neoplatonism

- Plotinus stressed that people did not have the mental capacity to fully understand both the ultimate reality itself or the consequences of its existence.
- Iamblichus distinguished further levels of reality in a hierarchy of levels beneath the ultimate reality.
- There was a level of reality corresponding to every distinct thought of which the human mind was capable.

Hypatia of Alexandria

- She is described by all commentators as a charismatic teacher.
- Hypatia came to symbolize learning and science which the early Christians identified with paganism.
- This led to Hypatia becoming the focal point of riots between Christians and non-Christians.

Hypatia of Alexandria

- She was murdered in March of 415 AD by Christians who felt threatened by her scholarship, learning, and depth of scientific knowledge.
- Many scholars departed soon after marking the beginning of the decline of Alexandria as a major center of ancient learning.
- There is no evidence that Hypatia undertook original mathematical research.

Hypatia of Alexandria

- However she assisted her father Theon of Alexandria in writing his eleven part commentary on Ptolemy's *Almagest.*
- She also assisted her father in producing a new version of Euclid's *Elements* which became the basis for all later editions.
- Hypatia wrote commentaries on Diophantus's *Arithmetica* and on Apollonius's *On Conics*.

Diophantus of Alexandria

- Often known as the "*Father of Algebra*".
- Best known for his *Arithmetica*, a work on the solution of algebraic equations and on the theory of numbers.
- However, essentially nothing is known of his life and there has been much debate regarding the date at which he lived.
- The *Arithmetica* is a collection of 130 problems giving numerical solutions of determinate equations (those with a unique solution), and indeterminate equations.

Diophantus of Alexandria

- A *Diophantine equation* is one which is to be solved for integer solutions only.
- The work considers the solution of many problems concerning linear and quadratic equations, but considers only positive rational solutions to these problems.
- There is no evidence to suggest that Diophantus realized that a quadratic equation could have two solutions.

Diophantus of Alexandria

- Diophantus looked at three types of quadratic equations $ax^2 + bx = c$, $ax^2 = bx + c$ and $ax^2 + c = bx$.
- He solved problems such as pairs of simultaneous quadratic equations.
- For example, consider $y + z = 10$, $yz = 9$.
  - Diophantus would solve this by creating a single quadratic equation in x.
  - Put $2x = y - z$ so, adding $y + z = 10$ and $y - z = 2x$, we have $y = 5 + x$, then subtracting them gives $z = 5 - x$.
  - Now $9 = yz = (5 + x)(5 - x) = 25 - x^2$, so $x^2 = 16$, $x = 4$
  - leading to $y = 9$, $z = 1$.

Diophantus of Alexandria

- Diophantus solves problems of finding values which make two linear expressions simultaneously into squares.
- For example, he shows how to find x to make $10x + 9$ and $5x + 4$ both squares (he finds $x = 28$).
- He solves problems such as finding x such that simultaneously $4x + 2$ is a cube and $2x + 1$ is a square (for which he easily finds the answer $x = 3/2$).

Diophantus of Alexandria

- Another type of problem is to find powers between given limits.
- For example, to find a square between 5/4 and 2 he multiplies both by 64, spots the square 100 between 80 and 128, so obtaining the solution 25/16 to the original problem.
- Diophantus also stated number theory results like:
  - *no number of the form $4n + 3$ or $4n - 1$ can be the sum of two squares;*
  - *a number of the form $24n + 7$ cannot be the sum of three squares.*

Arabic/Islamic Mathematics

- Research shows the debt that we owe to Arabic/Islamic mathematics.
- The mathematics studied today is far closer in style to that of the Arabic/Islamic contribution than to that of the Greeks.
- In addition to advancing mathematics, Arabic translations of Greek texts were made which preserved the Greek learning so that it was available to the Europeans at the beginning of the sixteenth century.

Arabic/Islamic Mathematics

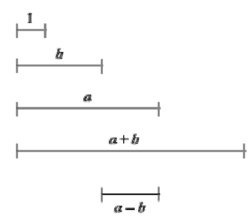
- A remarkable period of mathematical progress began with al-Khwarizmi's (ca. 780-850 AD) work and the translations of Greek texts.
- In the 9th century, Caliph al-Ma'mun set up the House of Wisdom (*Bayt al-Hikma*) in Baghdad which became the center for both the work of translating and of research.
- The most significant advances made by Arabic mathematics, namely the beginnings of algebra, began with al-Khwarizmi.

Arabic/Islamic Mathematics

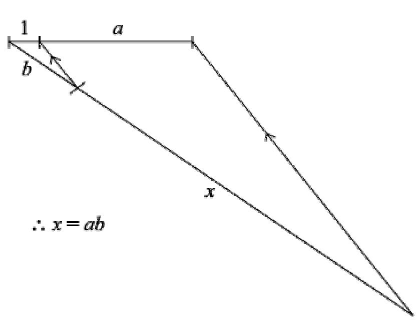
- It is important to understand just how significant this new idea was.
- It was a revolutionary move away from the Greek concept of mathematics which was essentially geometric.
- Algebra was a unifying theory which allowed rational numbers, irrational numbers, geometrical magnitudes, etc., to all be treated as "algebraic objects".

Geometric Constructions

- Euclid represented numbers as line segments.
- From two segments a, b, and a unit length, it is possible to construct $a + b$, $a - b$, $a \times b$, $a \div b$, a^2, and the square root of a.



Geometric Construction of *ab*



Arabic/Islamic Mathematics

- Algebra gave mathematics a whole new development path so much broader in concept to that which had existed before, and provided a vehicle for future development of the subject.
- Another important aspect of the introduction of algebraic ideas was that it allowed mathematics to be applied to itself in a way which had not happened before.
- All of this was done despite not using symbols.

Arabic/Islamic Mathematics

- Although many people were involved in the development of algebra as we know it today, we will mention the following important figures in the history of mathematics.
  - Mohammed Ibn Musa Al Khwarizmi
  - Thabit ibn Qurra
  - Abu Kamil
  - Omar Khayyam

Mohammed Ibn Musa Al Khwarizmi (ca. 780-850 AD)

- Sometimes called the "*Father of Algebra*".
- His most important work entitled *Hisab al-jabr w'al-muqabala* was written in 830.
- The word algebra we use today comes from *al-jabr* in the title.

Mohammed Ibn Musa Al Khwarizmi (ca. 780-850 AD)

- The word *al-jabr* means "restoring", "reunion", or "completion" which is the process of transferring negative terms from one side of an equation to the other.
- The word *al-muqabala* means "reduction" or "balancing" which is the process of combining like terms on the same side into a single term or the cancellation of like terms on opposite sides of an equation.

Mohammed Ibn Musa Al Khwarizmi (ca. 780-850 AD)

- He classified the solution of quadratic equations and gave geometric proofs for completing the square.
- This early Arabic algebra was still at the primitive rhetorical stage – No symbols were used and no negative or zero coefficients were allowed.
- He divided quadratic equations into three cases $x^2 + ax = b$, $x^2 + b = ax$, and $x^2 = ax + b$ with only positive coefficients.

Mohammed Ibn Musa Al Khwarizmi (ca. 780-850 AD)

- Solve $x^2 + 10x = 39$.
- Construct a square having sides of length x to represent x^2.
- Then add $10x$ to the x^2, by dividing it into 4 parts each representing $10x/4$.
- Add the 4 little $10/4 \times 10/4$ squares, to make a larger $x + 10/2$ side square.

Completing the Square

- By computing the area of the square in two ways and equating the results we get the top equation at the right.
- Substituting the original equation and using the fact that $a^2 = b$ implies $a = \sqrt{b}$.

$$\left(x+\frac{10}{2}\right)^2 = \left(x^2+10x\right)+4\left(\frac{10}{4}\right)^2$$
$$= 39 + \left(\frac{10}{2}\right)^2$$
$$= 39 + 25 = 64$$
$$\Rightarrow x + \frac{10}{2} = 8$$

Mohammed Ibn Musa Al Khwarizmi (ca. 780-850 AD)

- Also wrote on Hindu-Arabic numerals, in *Algoritmi de numero Indorum,* in English Al-Khwarizmi on the Hindu Art of Reckoning which gives us the word algorithm.
- The work describes the Hindu place-value system of numerals based on 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0.
- The first use of zero as a place holder in positional base notation was probably due to al-Khwarizmi in this work.

Al-Sabi Thabit ibn Qurra al-Harrani (826-901)

- Returning to the House of Wisdom in Baghdad in the 9th century, Thabit ibn Qurra was educated there by the Banu Musa brothers.
- Thabit ibn Qurra made many contributions to mathematics.

Al-Sabi Thabit ibn Qurra al-Harrani (826-901)

- He discovered a beautiful theorem which allowed pairs of amicable (“friendly”) numbers to be found, that is two numbers such that each is the sum of the proper divisors of the other.
- Theorem: For $n > 1$, let $p = 3\cdot 2^n - 1$, $q = 3\cdot 2^{n-1} - 1$, and $r = 9\cdot 2^{2n-1} - 1$. If p, q, and r are prime numbers, then $a = 2^n pq$ and $b = 2^n r$ are amicable pairs.

Al-Sabi Thabit ibn Qurra al-Harrani (826-901)

- He also generalized Pythagoras’ Theorem to an arbitrary triangle.
- Theorem: From the vertex A of ΔABC, construct B' and C' so that

$$\angle AB'C = \angle AC'C = \angle A.$$

Then $|AB|^2 + |AC|^2 = |BC|(|BB'| + |C'C|)$.

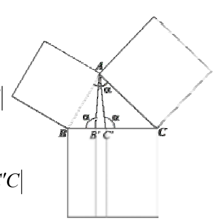
Proof of his Generalized Pythagorean Theorem

$$\frac{|AB|}{|BC|} = \frac{|B'B|}{|AB|} \Rightarrow |AB|^2 = |BC||B'B|$$

$$\frac{|AC|}{|BC|} = \frac{|C'C|}{|AC|} \Rightarrow |AC|^2 = |BC||C'C|$$

Adding the two equations gives

$$|AB|^2 + |AC|^2 = |BC||B'B| + |BC||C'C|$$



Abu Kamil Shuja ibn Aslam ibn Muhammad ibn Shuja (c. 850-930)

- Abu Kamil is sometimes known as *al-Hasib al-Misri*, meaning “the calculator from Egypt.”
- His *Book on algebra* is in three parts: (i) On the solution of quadratic equations, (ii) On applications of algebra to the regular pentagon and decagon, and (iii) On Diophantine equations and problems of recreational mathematics.
- The importance of Abu Kamil's work is that it became the basis for Fibonacci's book *Liber abaci*.

Abu Kamil Shuja ibn Aslam ibn Muhammad ibn Shuja (c. 850-930)

- Abu Kamil's *Book on algebra* took an important step forward.
- His showed that he was capable of working with higher powers of the unknown than x^2.
- These powers were not given in symbols but were written in words, but the naming convention of the powers demonstrates that Abu Kamil had begun to understand what we would write in symbols as $x^n x^m = x^{n+m}$.

Abu Kamil (c. 850-930)

- For example, he used the expression "square square root" for x^5 (that is, $x^2 \cdot x^2 \cdot x$), "cube cube" for x^6 (i.e. $x^3 \cdot x^3$), and "square square square square" for x^8 (i.e. $x^2 \cdot x^2 \cdot x^2 \cdot x^2$).
- His *Book on algebra* contained 69 problems.
- Let's look at an example!

Abu Kamil (c. 850-930)

- "*Divide 10 into two parts in such a way that when each of the two parts is divided by the other their sum will be 4.25*."
- Today we would solve the simultaneous equations $x + y = 10$ and $x/y + y/x = 4.25$.
- He used a method similar to the old Babylonian procedure.

Abu Kamil (c. 850-930)

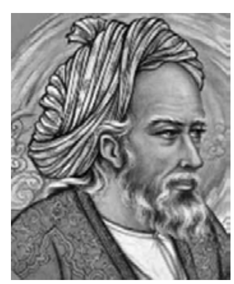
- Introduce a new variable z, and write
$$x = 5 - z \text{ and } y = 5 + z$$
- Substitute these into
$$x^2 + y^2 = 4.25xy$$
- Perform the necessary "restoring" and "reduction" to get
$$50 + 2z^2 = 4.25(25 - z^2)$$
- which is $z^2 = 9 \Rightarrow z = 3$.

Abu Kamil (c. 850-930)

- Abu Kamil also developed a calculus of radicals that is amazing to say the least.
- He could add and subtract radicals using the formula
$$\sqrt{a} \pm \sqrt{b} = \sqrt{a + b \pm 2\sqrt{ab}}$$
- This was a major advance in the use of irrational coefficients in indeterminate equations.

Omar Khayyam (1048-1131)

- Omar Khayyam's full name was *Abu al-Fath Omar ben Ibrahim al-Khayyam*.
- A literal translation of his name means "tent maker" and this may have been his fathers trade.
- Studied philosophy.



Omar Khayyam

- Khayyam was an outstanding mathematician and astronomer.
- He gave a complete classification of cubic equations with geometric solutions found by means of intersecting conic sections (a parabola with a circle).
- At least in part, these methods had been described by earlier authors such as Abu al-Jud.

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Omar Khayyam

- He combined the use of trigonometry and approximation theory to provide methods of solving algebraic equations by geometrical means.
- He wrote three books, one on arithmetic, entitled *Problems of Arithmetic*, one on music, and one on algebra, all before he was 25 years old.
- He also measured the length of the year to be 365.24219858156 days.

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Omar Khayyam

- Khayyam was a poet as well as a mathematician.
- Khayyam is best known as a result of Edward Fitzgerald's popular translation in 1859 of nearly 600 short four line poems, the *Rubaiyat*.
- Of all the verses, the best known is:

  The Moving Finger writes, and, having writ,
  Moves on: nor all thy Piety nor Wit
  Shall lure it back to cancel half a Line,
  Nor all thy Tears wash out a Word of it.

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Arabic/Islamic Mathematics

- Wilson's theorem, namely that if p is prime then $1+(p-1)!$ is divisible by p was first stated by Al-Haytham (965-1040).
- Although the Arabic mathematicians are most famed for their work on algebra, number theory and number systems, they also made considerable contributions to geometry, trigonometry and mathematical astronomy.

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Arabic/Islamic Mathematics

- Arabic mathematicians, in particular al-Haytham, studied optics and investigated the optical properties of mirrors made from conic sections.
- Astronomy, time-keeping and geography provided other motivations for geometrical and trigonometrical research.
- Thabit ibn Qurra undertook both theoretical and observational work in astronomy.
- Many of the Arabic mathematicians produced tables of trigonometric functions as part of their studies of astronomy.

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Indian Mathematics

- Mathematics today owes a huge debt to the outstanding contributions made by Indian mathematicians.
- The “huge debt” is the beautiful number system invented by the Indians which we use today.
- They used algebra to solve geometric problems.

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Indian Mathematics

- Although many people were involved in the development of the mathematics of India, we will discuss the mathematics of:
  - Aryabhatta the Elder (476-550)
  - Brahmagupta (598-670)
  - Bhaskara II (1114-1185)

Aryabhatta the Elder (476-550)

- He wrote *Aryabhatiya* which he finished in 499.
- It gives a summary of Hindu mathematics up to that time.
- It covers arithmetic, algebra, plane trigonometry and spherical trigonometry.
- It also contains continued fractions, quadratic equations, sums of power series and a table of sines.

Aryabhatta the Elder (476-550)

- Aryabhatta estimated the value of π to be 62832/20000 = 3.1416, which is very accurate, but he preferred to use the square root of 10 to approximate π.
- Aryabhata gives a systematic treatment of the position of the planets in space.
- He gave 62,832 miles as the circumference of the earth, which is an excellent approximation.
- Incredibly he believed that the orbits of the planets are ellipses.

Brahmagupta (598-670)

- He wrote *Brahmasphuta siddhanta* (The Opening of the Universe) in 628.
- He defined zero as the result of subtracting a number from itself.
- He also gave arithmetical rules in terms of fortunes (positive numbers) and debts (negative numbers).
- He presents three methods of multiplication.

Brahmagupta (598-670)

- Brahmagupta also solves quadratic indeterminate solutions of the form

$$ax^2 + c = y^2 \text{ and } ax^2 - c = y^2$$

- For example, he solves $8x^2 + 1 = y^2$ obtaining the solutions $(x,y) = (1,3), (6,17), (35,99), (204,577), (1189,3363), \ldots$
- Brahmagupta gave formulas for the area of a cyclic quadrilateral and for the lengths of the diagonals in terms of the sides.

Brahmagupta (598-670)

- Brahmagupta's formula for the area of a cyclic quadrilateral (i.e., a simple quadrilateral that is inscribed in a circle) with sides of length a, b, c, and d as

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

- where s is the semiperimeter (i.e., one-half the perimeter.)

Bhaskara (1114-1185)

- Also known as Bhaskaracharya, this latter name meaning "*Bhaskara the Teacher*".
- He was greatly influenced by Brahmagupta's work.
- Among his works were:
  - *Lilavati* (The Beautiful) which is on mathematics
  - *Bijaganita* (Seed Counting or Root Extraction) which is on algebra
  - the two part *Siddhantasiromani*, the first part is on mathematical astronomy and the second on the sphere.

Bhaskara (1114-1185)

- Bhaskara studied Pell's equation
  $px^2 + 1 = y^2$ for $p = 8, 11, 32, 61$ and 67.
- When $p = 61$ he found the solution
  $x = 226153980, y = 1776319049$.
- When $p = 67$ he found the solution
  $x = 5967, y = 48842$.
- He studied many of Diophantus' problems.

Bhaskara (1114-1185)

- Bhaskara was interested in trigonometry for its own sake rather than a tool for calculation.
- Among the many interesting results given by Bhaskara are the sine for the sum and difference of two angles, i.e.,
  $\sin(a + b) = \sin a \cos b + \cos a \sin b$
- and
  $\sin(a - b) = \sin a \cos b - \cos a \sin b$.

Roman Numerals

- In stark contrast to the Islamic and Indian mathematicians and merchants, their European counterparts were using clumsy Roman numerals which you still see today.
- Romans didn't have a symbol for zero.
- Sometimes numeral placement within a number can indicates subtraction rather than addition.

Roman Numerals

NUMBER	SYMBOL
1	I
5	V
10	X
50	L
100	C
500	D

NUMBER	SYMBOL
1,000	M
5,000	|⊃⊃
10,000	⊂⊂|⊃⊃
50,000	|⊃⊃⊃
100,000	⊂⊂⊂|⊃⊃⊃

Roman Numeral Examples

Hindu Arabic Numerals	Roman Numerals
87	LXXXVII
369	CCCLXIX
448	CDXLVIII
2,573	MMDLXXIII
4,949	M | ⊃⊃ CMXLIX
3,878	MMMDCCCLXXVIII
72,608	?

Roman Numerals

- Later, they introduced a horizontal line over them to indicate larger numbers.
- One line for thousands and two lines for millions.
- For example, 72,487,963 would be written as

$$\overline{\overline{\text{LXXII}}}\ \overline{\text{CDLXXXVII}}\ \text{CMLXIII}$$

Leonardo of Pisa, also called Fibonacci

- His book *Liber Abaci* was the first to introduce European to introduce the brilliant Hindu Arabic numerals.
- Imagine doing math with Roman numerals.
- MMMCMXCVII – MCMXCVIII = MCMXCIX.
- Which is 3997 – 1998 = 1999.

Brahmagupta’s Multiplication

- Let’s look at an example of what Brahmagupta called “*gomutrika*” which translates to “*like the trajectory of a cow’s urine*”
- Multiply: 235 × 284

2	2	3	5		
8		2	3	5	
4			2	3	5
	4	7	0		
	1	8	8	0	
			9	4	0
	6	6	7	4	0

Bhaskara’s Multiplication

- Bhaskara gave two methods of multiplication in his *Lilavati*. Let’s look at one of them.
- Multiply: 235 × 284

2	8	4	2	8	4		2	8	4
		2			3				5
4		8	6	1	2	1	0	2	0
1	6		2	4				4	0
5	6	8	8	5	2	1	4	2	0

Bhaskara’s Multiplication

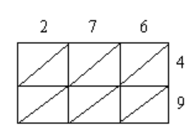
1	4	2	0
	8	5	2
	5	6	8
2	8	4	0

	1	4	2	0
	8	5	2	
5	6	8		
6	6	7	4	0

- Now, how do we add these products?
- Is the answer 2840 or 66740?
- Notice that in performing these multiplication, you need to be careful about lining up the digits.

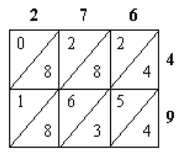
Hindu Lattice Multiplication

- To help keep the numerals in line, they used what is called Gelosia multiplication.
- For example, to multiply 276 × 49, first set up the grid as shown at the right.

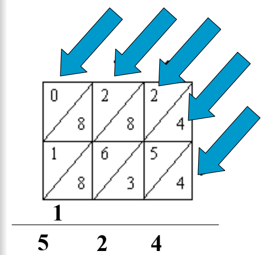


Lattice Multiplication

- Multiply each digit by each digit and place the resulting products in the appropriate square.
- Be sure to place the tens digits, if there is one, above the diagonal and the ones digit below.

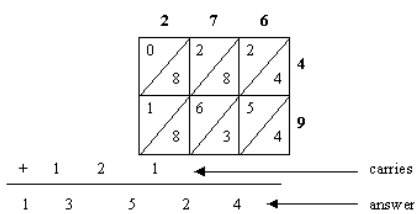


Lattice Multiplication



- Finally, sum the numbers in each diagonal and enter the total on the bottom.
- If the sum of the diagonal results in a two digit number, you need to carry as you would normally.

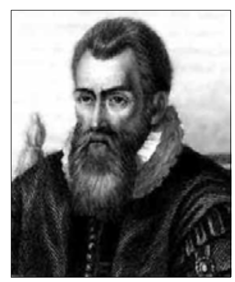
Hindu Lattice Multiplication



The answer is 13,524.

John Napier (1550-1617)

- A Scotsman famous for inventing logarithms.
- He used this lattice multiplication to construct a series of rods to help with long multiplication.
- The rods are called "*Napier's Bones*".



Chinese Mathematics

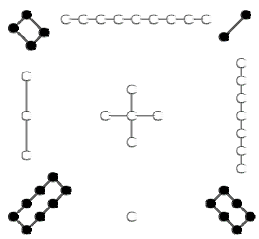
- The Chinese believed that numbers had "philosophical and metaphysical properties."
- They used numbers "to achieve spiritual harmony with the cosmos."
- The *ying* and *yang*, a philosophical representation of harmony, show up in the *I-Ching*, or *book of permutations*.
- The *Liang I* , or "two principles" are
  - the male *yang* represented by "——"
  - the female *ying* by "— —"

Chinese Mathematics

- Together they are used to represent the *Sz' Siang* or "four figures" and the *Pa-kua* or eight trigrams.
- These figures can be seen as representations in the binary number system if ying is considered to be zero and yang is considered to be one.
- With this in mind, the *Pa-kua* represents the numbers 0, 1, 2, 3, 4, 5, 6, and 7.
- The *I-Ching* states that the *Pa-kua* were footsteps of a dragon horse which appeared on a river bank.

The Lo-Shu

- Emperor Yu (c. 2200 B.C.) was standing on the bank of the Yellow river when a tortoise appeared with a mystic symbol on its back.
- This figure came to be known as the *lo-shu*.
- It is a magic square.



The Lo-Shu Magic Square

- The *lo-shu* represents a 3 × 3 square of numbers, arranged so that the sum of the numbers in any row, column, or diagonal is 15.
- In the 9th century, magic squares were used by Arabian astrologers to read horoscopes.

4	9	2
3	5	7
8	1	6

Magic Squares

- A *magic square* is a square array of numbers 1, 2, 3, ... , n^2 arranged in such a way that the sum of each row, each column and both diagonals is constant.
- The number n is called the *order* of the magic square and the constant is called the *magic sum*.
- In 1460, Emmanuel Moschopulus discovered the mathematical theory behind magic squares.

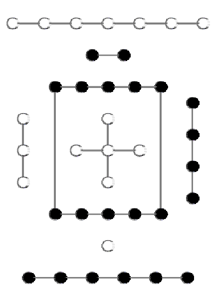
Magic Squares

- He determined that the *magic sum* is
  $(n^3 + n)/2$.
- The table at the right gives the first few values for a given n.
- Can you construct a 4×4 magic square whose sum is 34?

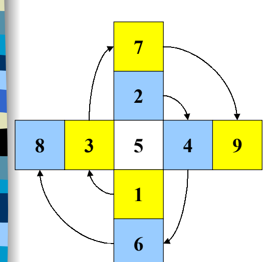
n	*Magic Sum*
3	*15*
4	*34*
5	*65*
6	*111*
7	*175*
8	*260*

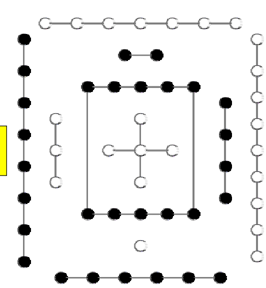
The Ho-t'u

- The Ho-t'u represents the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10.
- It was also a highly honored mystic symbol.
- Discarding the 5 and 10 both the odd and even sets add up to 20.



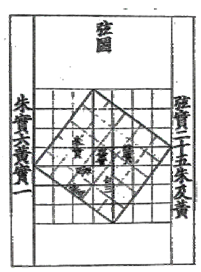
The Ho-t'u





Chinese Mathematics

- The *Zhoubi suanjing* (*Arithmetical Classic of the Gnomon and the Circular Paths to Heaven*) is one the oldest Chinese mathematical work.
- It contains a proof of the Pythagorean Theorem.



Chinese Mathematics

- *Jiuzhang suanshu* (*The Nine Chapters on the Mathematical Art*) is the greatest of the Chinese classics in mathematics.
- It consists of 246 problems separated into nine chapters.
- The problems deal with practical math for use in daily life.
- It contains problems involving the calculations of areas of all kinds of shapes, and volumes of various vessels and dams.

The Nine Chapters on the Mathematical Art

1. *Fang tian* – "Field measurement"
2. *Su mi* – "Cereals" is concerned with proportions
3. *Cui fen* – "Distribution by proportions"
4. *Shao guang* – "What width?"
5. *Shang gong* – "Construction consultations"
6. *Jun shu* – "Fair taxes"
7. *Ying bu zu* – "Excess and deficiency"
8. *Fang cheng* – "Rectangular arrays"
9. *Gongu* – Pythagorean theorem

Liu Hui

- Best known Chinese mathematician of the 3rd century.
- In 263, the he wrote a commentary on the *Nine Chapters* in which he verified theoretically the solution procedures, and added some problems of his own.
- He approximated π by approximating circles with polygons, doubling the number of sides to get better approximations.

Liu Hui

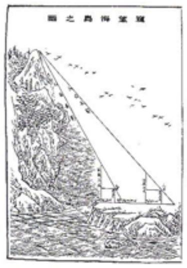
- From 96 and 192 sided polygons, he approximates π as 3.141014 and suggested 3.14 as a practical approximation.
- He also presents Gauss-Jordan elimination and Calvalieri's principle to find the volume of cylinder.
- Around 600, his work was separated out and published as the *Haidao suanjing* (*Sea Island Mathematical Manual*).

Sea Island Mathematical Manual

- It consists of a series of problems about a mythical Sea Island.
- It includes nine surveying problems involving indirect observations.
- It describes a range of surveying and map-making techniques which are a precursor of trigonometry, but using only properties of similar triangles, the area formula and so on.

Sea Island Mathematical Manual

- Used poles with bars fixed at right angles for measuring distances.
- Used similar triangles to relay proportions.



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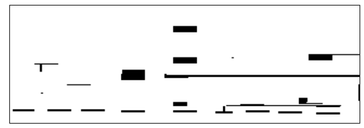
Chinese Stick Numerals



- Originated with bamboo sticks laid out on a flat board.
- The system is essentially positional, based on a ten scale, with blanks where zero is located.

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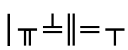
Chinese Stick Numerals

- There are two sets of symbols for the digits 1, 2, 3, …, 9, which are used in alternate positions, the top was used for ones, hundreds, etc, and the bottom for tens, thousands, etc.
- Eventually, they introduced a circle for zero.
- For example, 177,226 would be 

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Traditional Chinese Numbers

Number	*Symbol*	*Number*	*Symbol*	*Number*	*Symbol*
1	一	6	六	10	十
2	二	7	七	100	百
3	三	8	八	1,000	千
4	四	9	九	10,000	万
5	五	***Chinese Numbers***			

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Traditional Chinese Numbers

- Symbols for 1 to 9 are used in conjunction with the base 10 symbols through multiplication to form a number.
- The numbers were written vertically.
- Example:
  - The number 2,465 appears to the right.

二千四百六十五

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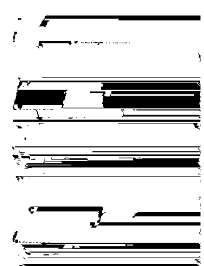
The Mayans

- The classic period of the Maya spans the period from 250 AD to 900 AD, but this classic period was built atop of a civilization which had lived in the region from about 2000 BC.



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The Mayans

- Built large cities which included temples, palaces, shrines, wood and thatch houses, terraces, causeways, plazas and huge reservoirs for storing rainwater.
- The rulers were astronomer priests who lived in the cities who controlled the people with their religious instructions.
- A common culture, calendar, and mythology held the civilization together and astronomy played an important part in the religion which underlay the whole life of the people.

The Mayans

- *The Dresden Codex* is a Mayan treatise on astronomy.
- Of course astronomy and calendar calculations require mathematics .
- The Maya constructed a very sophisticated number system.
- We do not know the date of these mathematical achievements but it seems certain that when the system was devised it contained features which were more advanced than any other in the world at the time.

Mayan Numerals

- The Mayan Indians of Central America developed a positional system using base 20 (or the vegesimal system).
- Like Babylonians, they used a simple (additive) grouping system for numbers 1 to 19.
- They used a dot (•) for 1 and a bar (—) for 5.

Mayan Numerals

1	•	6	• —	11	• — —	16	• — — —
2	••	7	•• —	12	•• — —	17	•• — — —
3	•••	8	••• —	13	••• — —	18	••• — — —
4	••••	9	•••• —	14	•••• — —	19	•••• — — —
5	—	10	— —	15	— — —		

Mayan Numerals

- The Mayan year was divided into 18 months of 20 days each, with 5 extra holidays added to fill the difference between this and the solar year.
- Numerals were written vertically with the larger units above.
- A place holder ( ) was used for missing positions. Thus, giving them a symbol for zero.

Mayan Numerals

- Consider the number at the right along with the computation converting it into our number system.
- Note the number is written vertically.
- With the high base of twenty, they could write large numbers with a few symbols.

•••• —	=	9×20^4	=	1440000
	=	0×20^3	=	0
• —	=	6×20^2	=	2400
••• — —	=	13×20^1	=	260
••	=	2×20^0	=	2
				1442662

Mayan Calendars

- The Maya had two calendars.
- A ritual calendar, known as the *Tzolkin*, composed of 260 days.
- A 365-day civil calendar called the *Haab*.
- The two calendars would return to the same cycle after $lcm(260, 365) = 18980$ days.

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